PILE LAYOUT TO MINIMIZE INTERFERENCE
SUMMARY OF DETERMINISTIC AND PROBABILISTIC METHODS

1. Construction Specifications and Tolerances

The theoretical ground location for piles is normally shown on construction drawings and laid out in the field by a survey crew. As it is impractical to drive a pile at precisely the theoretical location, specifications provide for some tolerance or deviation between the theoretical location and actual location. A commonly specified tolerance (References a, b, e, f, and h) is 3 in., although deviations as great as 6 in. are sometimes allowed for timber piles, where the nonuniform shape makes it difficult to hold the pile in position during driving. Likewise, it is impractical to drive a pile precisely vertical or precisely at the specified angle or batter. Specifications usually allow a deviation from vertical or from the theoretical batter in the range of 0.15 to 0.50 in./ft, with 0.25 in./ft (Reference b) being a common value.

2. Spacing to Avoid Interference -- Deterministic Solution

Assume that two adjacent piles of length L are driven at the maximum permissible deviation, Δx_{max} , from the specified location and are inclined at the maximum permissible deviation from plumb, Δp_{max} . If these deviations are combined in the most unfavorable directions, as shown in Figure 1-1, a minimum pile spacing can be determined that will ensure that no intersections occur, providing that the piles are in fact driven within the tolerances. Neglecting the minor difference in apparent pile diameter due to the pile inclination, the resulting minimum center-to-center pile spacing, AX_{min} , is:

$$AX_{min} = 2 \text{ piles} * 1 \text{ ft}/12 \text{ in.} * [\Delta x_{max} + (L\Delta p_{max}) + D/2]$$

or

$$AX_{min} = (1/6)(\Delta x_{max} + L\Delta p_{max} + D/2)$$

where

AX_{min} is the minimum allowable center-to-center pile spacing in feet

Δx_{max} is the maximum permissible ground location error in inches (typically 3 inches)

Δp_{max} is the maximum permissible inclination error in inches per foot (typically 0.25 in./ft)

L is the pile length in feet

D is the pile diameter or width in inches

Where AX_{min} is less than the specified pile spacing, pile intersection can only occur if the piles are driven out of tolerance, and no further studies are necessary.

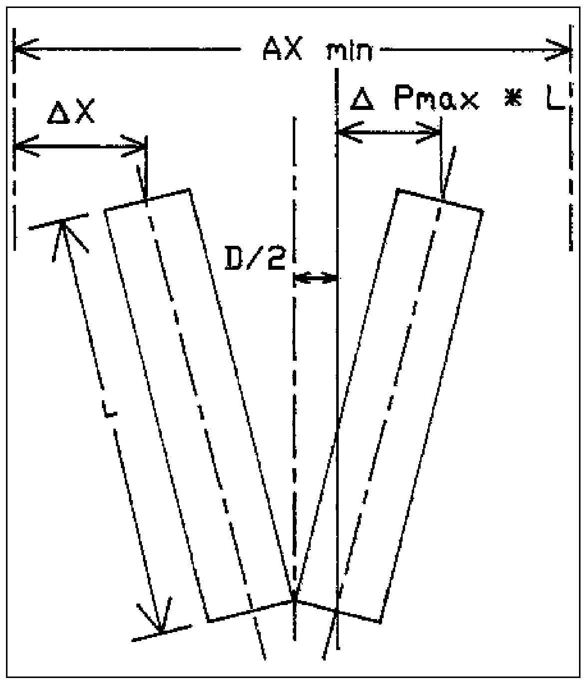

Figure 1-1. Deterministic solution for minimum pile spacing

3. Spacing to Avoid Intersection -- Probabilistic Solution

For a pile layout consisting of 14 in. diameter piles 100 ft long, the equation above gives a minimum spacing of 70 in. or 5.833 ft; hence piles spaced on 5 ft centers could theoretically intersect. However, the probability of one or more such intersections may be quite low, and perhaps tolerable. As shown in Figure 1-2, the location of a pile at any depth

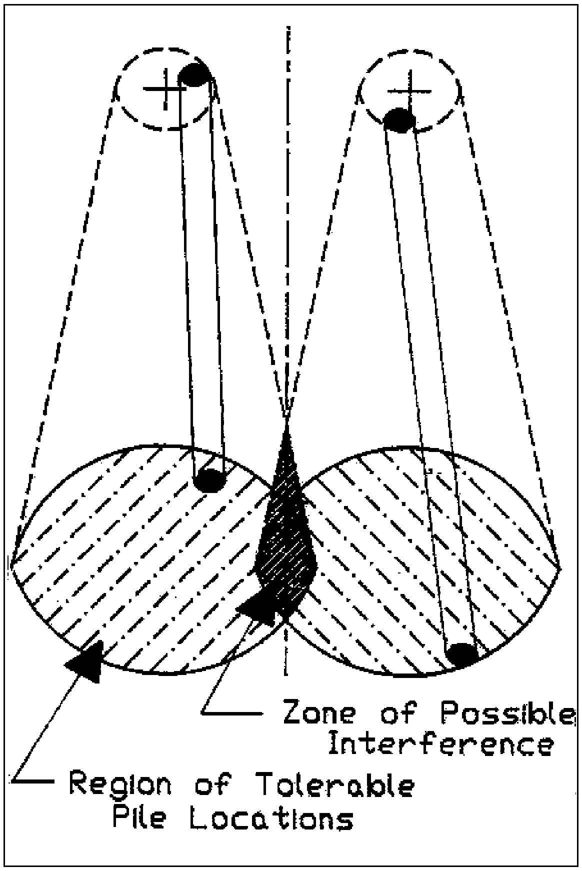

Figure 1-2. Basis of probabilistic solution

may be viewed as a two-dimensional area at a random location in a much larger area representing all the possible locations. Furthermore, certain locations, such as those corresponding to small deviations, are more probable than other locations, such as those corresponding to extreme deviations. For an intersection to occur, the pile location at some depth must overlap that of another pile, which is also random. The coincidence of two random locations overlapping at some depth has a probability which can be calculated, at least approximately. The resulting probability value can then be used to assess the probability of one or more intersections occurring in a group of a given size. A method and computer solution for assessing such probabilities have been developed (Reference j). The basis of the method is summarized below. A detailed user's guide for the program package, CPGP, is available (Reference f).

4. Estimating the Probability of Intersection for a Single Interior Pile

a. *Assumptions*. A typical interior pile in a large group is illustrated in Figure 1-3. The piles are assumed to be uniformly spaced at distances AX in the x direction and AY in the y direction. Furthermore, the piles are assumed to be round with diameter D and length L. As the intent of the analysis is to estimate the order of magnitude of the intersection probability rather than a precise value, rectangular piles and other shapes are modeled as equivalent round piles. The piles are assumed to be rigid; hence intersections are assumed to occur only from unfavorable location and alignment combinations, not from deflection by a boulder or similar obstruction.

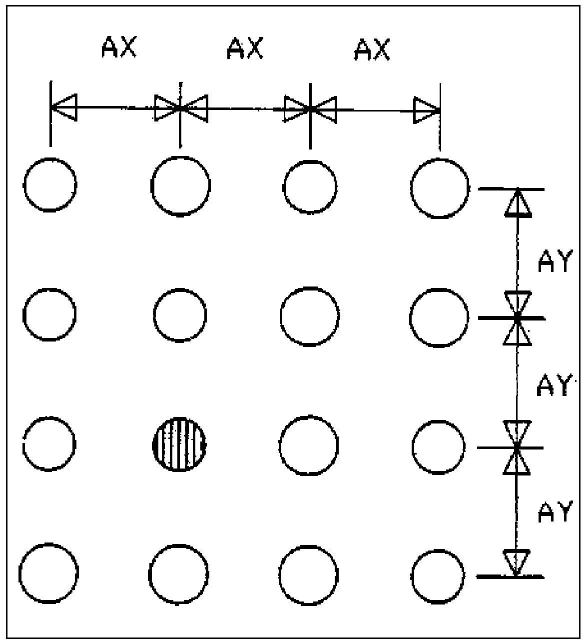

Figure 1-3. Typical interior pile in group

b. *Definition of variables*. The relationship of the theoretical pile location to the driven location and alignment are shown in Figure 1-4. The ground location of a driven pile is assumed to deviate from the theoretical location by two placement error components, Δx and Δy. The slope of the pile is assumed to deviate from the vertical or the theoretical batter by two alignment error components, the batter error, Δb, and the plumb error, Δp, where Δb and Δp are expressed in inches/foot. The batter error is defined in the plane of the crane boom travel, and the plumb error is defined perpendicular to the batter error. In the field, the batter error may

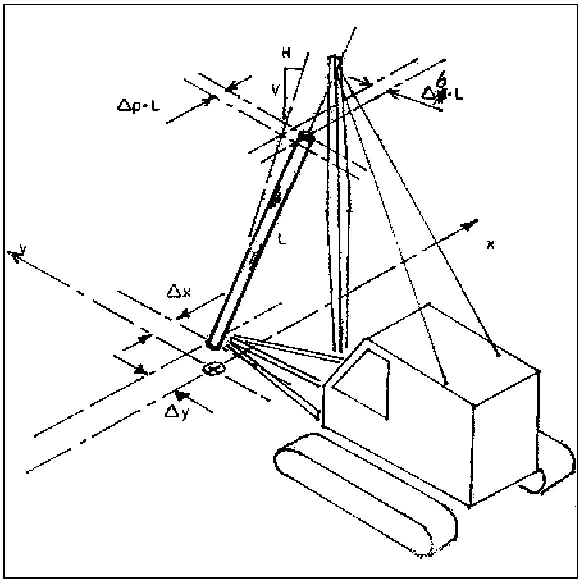

Figure 1-4. Definition of placement and alignment errors

be smaller than the plumb error as the crane operator has better control when aligning the pile and leads in this plane. The four error variables, Δx, Δy, Δb, and Δp are *random variables*. They cannot be assigned specific values as their values vary from pile to pile, but they can be defined in terms of a probability distribution. That is, probabilities can be associated with the value of the random variable being greater than or less than any particular value.

c. Probabilistic analysis. If one assumes that reasonable probability distributions can be defined for the four error variables, the probability that the axis of the pile will pass through any point in the ground can be determined. In concept, the probability that the pile will intersect another is determined by calculating the probability that the first pile will pass through a given point and a second pile will pass through the same point, and then integrating or summing over all such possible points and all possible second piles. The software package CPGP solves for the probability of intersection using a random number simulation or *Monte Carlo* analysis. Rather than perform complex integrations, the program repeatedly simulates the driving of a pile and eight surrounding piles, as shown in Figure 1-5. For each trial simulation, random values for the four error variables are generated for each of the nine piles, for a total of 36 random values. The axis locations of the piles are calculated, and a check is made whether the distance

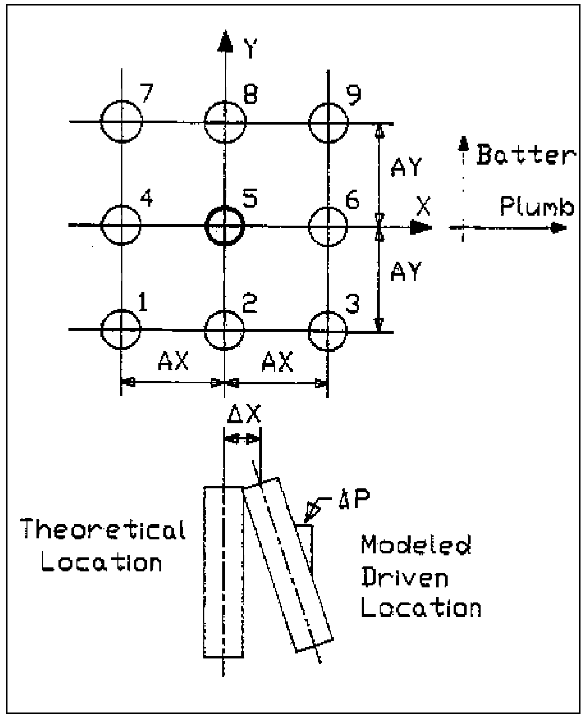

Figure 1-5. Typical nine-pile group for analysis

from the axis of the interior pile to the axis of any other pile is less than the pile diameter at any depth. If so, an intersection would occur for that particular combination of the 36 error values. The simulation is repeated for a large number of trials, each with newly generated random values for the error variables. The error values are generated such that, in the long run, the distribution of their values matches the assumed probability distributions. As the number of trials becomes large, the ratio of the number of trials with intersections to the total number of trials provides an increasingly accurate estimate of the probability of intersection.

d. Probability distribution for the error variables. The four random variables characterizing the pile placement and alignment errors, Δx, Δy, Δp, and Δb are assumed to fit the normal, or Gaussian, distribution found in most standard statistics books. This assumption is justifiable and convenient for a number of reasons:

(1) The normal distribution is bell-shaped and symmetrical. If the pile is assumed to be driven, on the average, at the theoretical location, then small deviations are more likely than large ones, and deviations are equally likely in either direction. These properties are consistent with expected construction practice.

(2) The normal distribution is commonly used to model random errors in a variety of systems; in fact, its development traces from error analysis.

(3) The normal distribution is completely defined by two parameters, the mean and standard deviations; if these are specified for the error variables, their entire distributions are defined and the probability of the variables assuming any set of values is readily calculated.

(4) Normally distributed random numbers are easily generated by simple computer algorithms.

The means of the four random error variables are taken as zero. This implies that, on the average, the piles are driven at the theoretical location and alignment. The standard deviations of the error variables, $\sigma_{\Delta x}$, $\sigma_{\Delta y}$, $\sigma_{\Delta b}$, and $\sigma_{\Delta p}$, define a measure of the scatter of the possible values about the mean. As the normal distribution extends to plus and minus infinity, the variables can assume any value; however, as illustrated in Figure 1-6, the values have a practical range of 3 to 4 standard deviations. For a normally distributed random variable, 68.27 percent of all values will lie within one standard deviation from the mean, 95.45 percent within 2 standard deviations, 99.73 percent within 3 standard deviations, and 99.994 percent within 4 standard deviations.

e. Default values for the standard deviation of the error variables. The software package CPGP provides default values for the standard deviation of the four error variables. If better data on the expected deviations are available, other values may be specified at the time of program execution. The standard deviations are used by the random number generator to scale the variation of the generated error values. The default values were selected to be consistent with both actual construction and normal tolerances. In a reasonably well-controlled manufacturing or production process aimed at producing products within a tolerance, the acceptable tolerances will typically correspond to bounds of two to three standard deviations from the mean value (Reference j). Assuming that this is the case and the pile deviations are normally distributed would imply

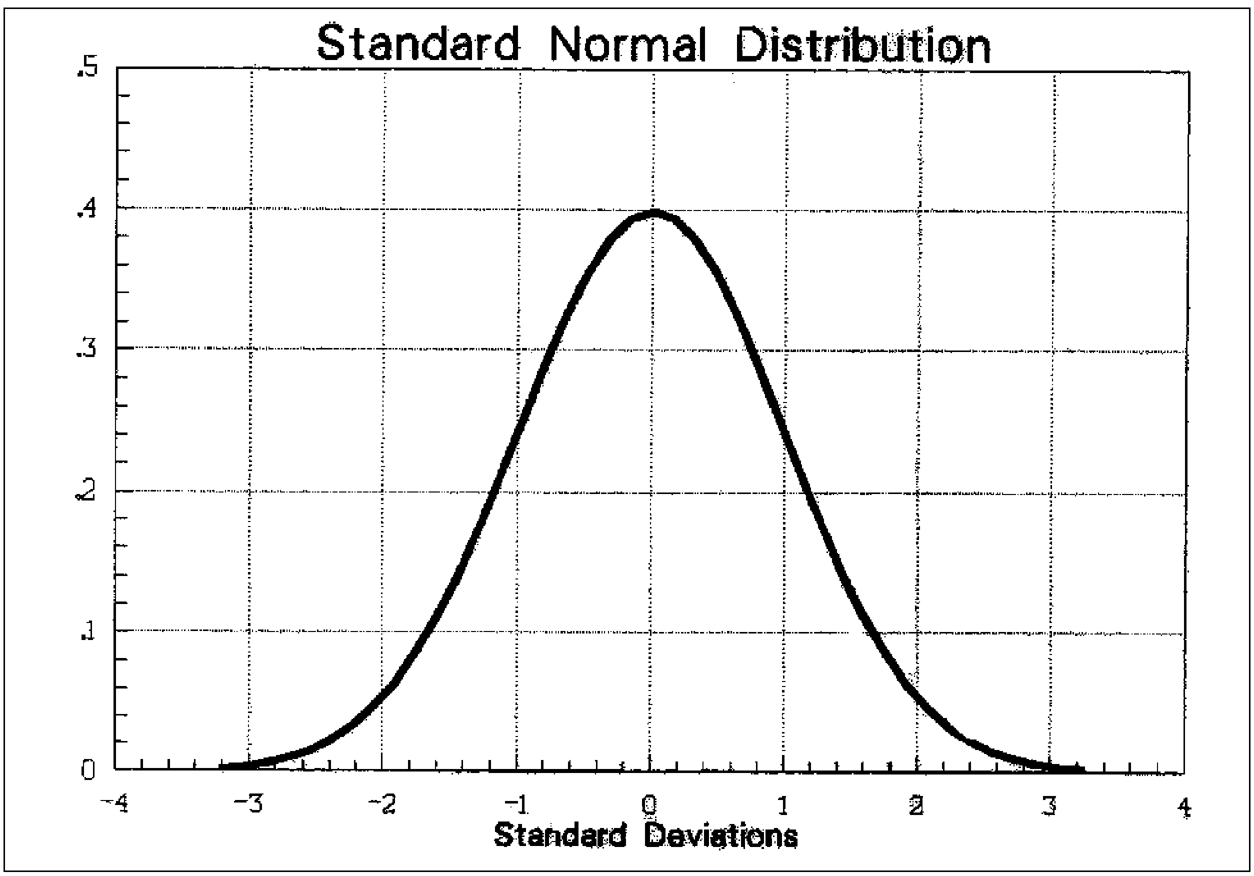

Figure 1-6. Standard normal distribution

that 4.55 percent (2σ) to 0.27 percent (3σ) of the piles will be driven out-of-tolerance and either rejected or erroneously approved, and specified tolerances should be met 95 to 99 percent of the time if "normal" pile driving practice is followed. To assign the default standard deviations for the error variables in the program, actual data on driven piles at Locks and Dam No. 26 (Replacement) were evaluated, and the standard deviations of the errors were compared to tolerances. The default values and their relation to usual tolerances are shown in Table 1-1. Using the above default values and tolerances, the program will generate location errors that exceed usual tolerance 4.55 percent of the time, batter errors that exceed usual tolerance 1.24 percent of the time, and plumb errors that exceed usual tolerance 9.70 percent of the time. If normal inspection procedures will detect and correct most cases where tolerances are exceeded, use of the default values should lead to a conservative estimate of intersection probability.

f. Program use. Detailed instructions for using the software package CPGP are contained in the user's guide (Reference h). The software package is written for IBM compatible PC's using the MS-DOS operating system and has the capability of modeling vertical or battered piles, and straight or cambered piles. The probability of intersection for a single interior pile is evaluated as a function of pile diameter, length, spacing, batter, and the standard deviations of the four error variables. Because of the iterative nature of the simulation program, and the fact that confidence limits on the solution are inversely proportional to the square root of the number of trials, running times can be relatively long (as much as two hours on an 80386 microcomputer with 80387 math coprocessor).

5. Chart Solutions

Due to the relatively long running time for the simulation program and its potentially infrequent use, a set of chart solutions has been prepared for vertical, uncambered piles using the default values for the random error variables and assuming that the pile spacing is equal in both directions. These charts are provided in Enclosure 3. For many cases, these charts will provide sufficient information to determine the probability of intersection for a single interior pile. Where piles are to be driven on a batter, or where different values for the standard deviations of the error variables are assumed, it is necessary to run the simulation.

6. Estimating the Probability Distribution for the Number of Intersections in a Pile Group

a. Equivalent number of interior piles. The probability of intersection for a typical interior pile is desired to obtain the expected number of intersections and the probability of 0, 1, 2, etc., intersections in a large group of piles. Special considerations must be made for the exterior and corner piles in a group. As an exterior pile has adjacent piles in only two of four quadrants, it is approximated by a statistically equivalent to one-half an interior. Likewise, the corner piles in a group are approximated by statistically equivalent of one-fourth an interior pile. While the approximations for the exterior and corners are not exact solution for probability distribution of these piles, they do provide sufficient accuracy for this application. Thus, the equivalent number of interior piles in a group can be taken as:

Table 1-1
Standard Deviation Versus Tolerance

Variable	Program Default Value for Standard Deviation	Usual Tolerance	Tolerance/Std. Dev.
Δx and Δy	$\sigma_{\Delta x}$=1.5 in.	3.0 in.	2.0
Δb	$\sigma_{\Delta b}$=0.10 in./ft	0.25 in./ft	2.5
Δp	$\sigma_{\Delta p}$=0.15 in./ft	0.25 in./ft	1.666

Interior piles + (1/2) exterior piles

+ (1/4) corner piles

For a group of m rows by n columns, this becomes:

$$(m-2)(n-2) + (1/2)(2)[(m-2)+(n-2)] + (1/4)(4)$$

Expanding and collecting terms, this becomes:

$$EIP = mn - m - n + 1$$

where EIP is the number of equivalent interior piles. For example, a 20 by 40 pile group would have 800 actual piles and 741 equivalent interior piles.

b. Expected number of intersections. The simulation model calculates the probability that the center pile of a nine pile group will intersect an adjacent pile. If a group of piles is considered, an intersection is not an independent event as every intersection involves two piles. However, a conservative estimate of the total probability of intersection can be obtained by assuming independence and employing the binomial distribution. This is analogous to assume that driving pile 18 into pile 21 is a different event than driving pile 21 into pile 18. In probability theory, the binomial distribution is used to predict the probability of the number of "successes" x, that will occur in a set of n independent trials when the probability of success is p for each trial. For the problem at hand, a pile intersection is considered a "success" in the probabilistic sense. The expected number of successes, or intersections, I, is given by:

$$E[I] = Np$$

where N is taken as the equivalent number of interior piles. Thus, for the example 20 x 40 pile group, if p has been previously found to be 0.002 for a single pile:

$$E[I] = (741)(.002) = 1.482$$

The expected value of 1.482 is the best estimate that can be made of the probable number of intersections. If a cost can be identified for the occurrence of an intersection, then the expected number of intersections times the cost per intersection represents the financial risk.

c. Probability distribution for the number of intersections. Although the expected number of intersections for this example is 1.482, the actual number of intersections must be a member of the set 0, 1, 2, ... According to the binomial distribution, the probability of x intersections is:

$$Pr(x) = \left(\frac{N!}{x!(N-x)!}\right) p^{x}(1-p)^{N-x}$$

Replacing x with I and continuing with the example, the binomial distribution gives:

Pr(I=0) = 0.22685Pr(I>0) = 1.0 - 0.22685
= 0.77315
Pr(I=1) = 0.33686Pr(I>1) = 1.0 - 0.22685
- 0.33686 = 0.43629
Pr(I=2) = 0.24978Pr(I>2) = 0.18831
Pr(I=3) = 0.12330Pr(I>3) = 0.06501
etc.

Thus, there is about a 23 percent chance of no intersections, a 77 percent chance of at least one intersection, a 43 percent chance of more than one intersection, a 19 percent chance of more than two intersections, and only a 6.5 percent chance of more than three intersections. Due to the factorials, the binomial distribution becomes unwieldy to calculate, even with a computer, for large values of N. It can be closely approximated using the Poisson distribution in the following form:

$$Pr(x) = \frac{(Np)^x}{x!} e^{-Np}$$

Again, x would be replaced with the number of intersections, I. The following tabulation indicates the similarity of the binomial and Poisson solutions for the case of a 20- by 40-pile group with p = 0.002.

No. of Intersections, I	Pr(I) (Binomial)	Pr(I) (Poisson)
0	0.22685	0.22718
1	0.33686	0.33669
2	0.24978	0.24948
3 or more	0.18651	0.18665

The software package CPGP provides a convenient means for calculating the distribution of the number of intersections using the Poisson distribution.

7. Limitations

The probabilistic approach to pile interference assessment is not a substitute for writing specifications that are as restrictive as necessary and enforcing them by adequate quality control and quality assurance procedures. In fact, the probabilistic procedure depends on such control and implies that the standard deviation of the actual alignment errors will not be greater than about one-third to one-half the specification tolerance. The procedure assumes rigid piles does not account for bending of battered piles that are inadequately supported or piles veering from a straight line due to obstructions.

8. Tolerable Probabilities

The use of a probabilistic procedure implies that some piles may intersect. If an intersection is believed to have occurred, the as-driven location and alignment should be measured to ensure that the specifications have been met. Then the piles should be pulled, inspected, replaced if necessary, and redriven. If one or more intersections go unnoticed, the intersecting piles may sustain structural damage and not provide the design capacity. For piles to be pulled, costs can be associated with pulling, redriving, and related delays. These costs can be multiplied by the expected number of intersections given in paragraph 6 to determine the expected intersection cost. The probability distribution for the expected intersection cost can be determined by multiplying the probability of 0, 1, 2, 3, etc., intersections by the associated costs of such intersections. As pile intersections may not always be apparent, a check should be made of the foundation capacity associated with 1, 2, 3, etc. random piles being damaged. In the absence of detailed cost and capacity studies, it would appear prudent to develop pile layouts such that the probability of intersection for single piles is less than about 0.002 and the expected number of intersections in a group is less than about 0.5.

9. Preliminary Findings

Experience with the probabilistic procedure is limited at this time. From the previous research, certain general conclusions can be drawn.

a. Pile length and spacing. The probability of intersection is sensitive to pile length. For common pile sizes and spacings, pile lengths shorter than 50 to 60 ft correspond to small probabilities of intersection, and lengths greater than 80 to 90 ft correspond to relatively large probabilities of intersection. Between these ranges, the probability of intersection increases two or more orders of magnitude.

b. Placement and alignment tolerances. The primary factor affecting pile intersection is the plumb and alignment tolerance, typically set at 0.25 in./ft. Variations in the standard deviation of the alignment error result in significant changes in the probability of intersection. Variations in the standard deviation of the ground placement error make much less difference. At 0.25 in./ft, the tip of a 100-ft long pile would deviate 25 in. from its theoretical location, which is over eight times greater than the 3 in. maximum deviation caused by ground placement. Thus, where intersection is of concern, efforts should be made to carefully inspect pile alignment in the field.

c. Pile camber and sweep. Standard specifications allow piles to deviate from perfect straightness (References c, d, h). The CPGP software allows the modeling of cambered and swept (curved) piles. The degree of camber and sweep on all simulated piles is taken to be that specified but the direction of curvature is simulated randomly. While it might be expected that cambered and swept piles would be more likely to intersect than straight piles, comparative analyses show a negligible difference. In a probabilistic model, curved piles are equally likely to curve away from each other as toward each other, and these effects tend to cancel out.

10. References

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